On mathematical work by Roin Nadiradze

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Mathematical work of Roin Nadiradze is in the domain of Algebraic Topology, basically in Cobordism Theory.

The concept of cobordism is very natural in mathematics. A cycle defined by a closed submanifold of dimention n is assumed to be equivalent to zero if there exists a submanifold of dimension $n+1$ such that the first manifold is its boundary. On such an intuitive level, Poincare gave the first definition of homology in Analysis situs (1895). The next step in development of cobordism theory was done by Pontryagin in 1938, who calculated the first two stable homotopy groups of spheres by the framed cobordism. This was the great Pontrygin's idea to connect cobordism and homotopy. Later this was developped by Thom in 1954. Pontryagin proved in 1947 that if two manifolds are bordant, then their characteristic numbers coincide. Non-oriented and oriented cobordisms were introduced in 1951-1953 by Rokhlin, who calculated these groups in small dimensions. Thom established for the oriented and non-oriented cobordisms connection between cobordisms and homotopy properties of certain complexes which were called later Thom complexes. Thom's work has made great progress in understanding and calculating cobordisms. Attention to cobordism was also drawn due to Markov's theorem (1958) on the algorithmic undecidability of the homeomorphism problem and even the homotopy equivalence of manifolds of dimension four and higher. Cobordism gives a weaker equivalence relation. After Thom's work, it became possible to define and study cobordisms of manifolds with various structures in the stable normal bundle. One of the most interesting cobordism theories is the cobordism theory of stably complex manifolds. The ring of stably complex cobordism was calculated by Milnor (1960) and Novikov (1962) and it is isomorphic to the polynomial ring over the integers on the infinite number of generators. One of the most remarkable features of the complex cobordism is the connection with the theory of formal groups. It was found by Mishchenko and Novikov in 1967 and it was developed by Quillen in his work of 1969, where he proved that the formal group of complex cobordism is isomorphic to universal formal group of Lazard. According to the Frobenius theorem, every finite-dimensional associative division algebra over the real numbers is isomorphic to one of the following: the real numbers, the complex numbers, the quaternions. This was one of the motivations to study the cobordism of stably quaternionic manifolds (symplectic cobordism). The question of study this cobordism was formally posed by Milnor in his work of 1960. First calculations were done by Novikov in 1962. It turned out that this ring is rather difficult to calculate, and its structure is still unknown (2021), despite numerous attempts to do this. In 1967 Stong constructed the series of stably quaternionic manifolds, which gives explicit mod p generators of the symplectic cobordism ring for odd primes p. In connection with the study of the natural homomorphism from symplectic cobordism to complex cobordism, Buchstaber and Novikov introduced two-valued formal groups in 1971. This concept was later generalized by Buchstaber, who constructed the theory of multivalued formal groups. Roin Nadiradze began his mathematical activity under the guidance of Novikov and Buchstaber. In this sense, he belongs to the great mathematical school and tradition: Pontryagin-Postnikov-Novikov-Buchstaber. This became possible largely thanks to the initiative of Academician Georgy

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Chogoshvili to ask the leadership of the Lomonosov Moscow State University to accept a group of gifted Georgian students. In turn, this tradition and contacts continue.

In his first article [1] R. Nadiradze studied the Stong manifolds and cobordism of self-conjugate manifolds. Stong manifolds $M(n, m)$ (as mentioned above) are the examples of manifolds whose stable normal bundle is a quaternionic vector bundle. They are defined as submanifolds of the product of complex projective spaces $\mathbf{CP}^{2n+1} \times \mathbf{CP}^{2m+1}$ dual to the quaternionic line bundle. In this situation complex projective spaces of odd complex dimension can be viewed as stably quaternionic manifolds. In this note [1] R. Nadiradze, described by equations the models for $M(n, m)$ as well as a fixed point free involution T on $M(n, m)$. Then he proved that the stable tangent bundle of $M(n, m)/T$ has a complex self-conjugate structure; this is a useful fact for the study of self-conjugate cobordism. In several subsequent articles [2, 3, 4, 5, 6, 7] Roin continued this study.

One of most interesting works by R. Nadiradze [11] is about elements of finite order in the symplectic cobordism ring Ω^*_{Sp}. He constructed geometrically and examined a new sequence of torsion elements in Ω^*_{Sp}. Among these elements there is one of dimension 111. It was proved latter by the Adams-Novikov spectral sequence, that there is an element of order 4 in Ω^*_{Sp} in this dimension. It is not proved yet whether it coincides with the Roin's element. He also conjectured that all torsion elements in Ω^*_{Sp} have order 2 or 4. This conjecture is still open.

In the work [17] R. Nadiradze constructed the series of geometrical generators and established relations in the self-conjugate cobordism theory SC^*. Although this series do not form a complete system, they give interesting information about SC^* cobordism ring.

In the articles [8, 10] Roin clarified that the classical three properties of characteristic classes are inconsistent in SC-theory. Of course, it was decided to give up the dimensions property and to introduce negative characteristic classes in SC^*-theory in order to keep the Whithey formula and the Euler class as top class. Therefore in contrast to Stiefel-Whithey, Chern or Pontryagin classes, negative characteristic classes in SC^* theory are of dimensions $-\infty, \cdots, n$. These negative classes give undecomposable elements in SC^*-cobordism. Some of these classes were realized geometrically in [15, 17].

In his Thesis Roin Nadiradze also studied the formal group laws [9, 12, 14] and their connection with cobordism theory. In [13, 16, 18] he introduced a class of formal group laws, which now is called the Nadiradze formal group laws. It was worked out later [19, 20] that the Nadiradze formal group laws give another derivation of the Buchstaber and Krichever formal group laws and related genera.

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